

Practice Test

Chapter 3 – Set Theory & logic

Name: _____

Block: _____

Multiple Choice

Identify the choice that best completes the statement or answers the question.

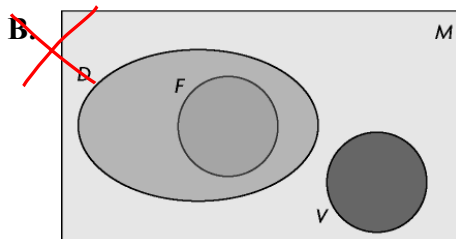
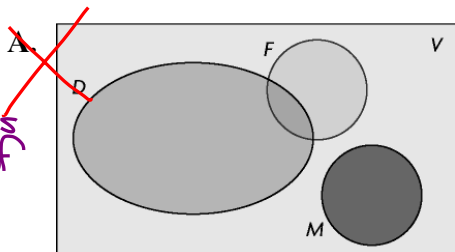
D

1. Which Venn diagram correctly represents the situation described?
Rahim described the set as follows:

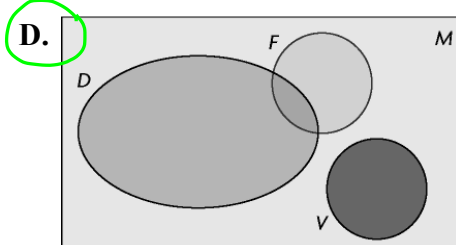
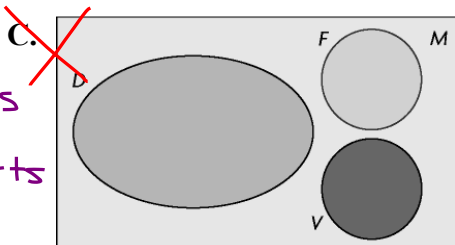
- $M = \{\text{all of the foods he eats}\}$
- $D = \{\text{his favourite desserts}\}$
- $V = \{\text{his favourite vegetables}\}$
- $F = \{\text{his favourite fruits}\}$

Assume Rahim likes some fruit for dessert.

This means that the fruit & dessert subsets should overlap.



Not this one. Only some fruits are desserts.



This one says All foods is a subset of vegetables.

This one has no overlap of fruit & desserts

B

2. What is true about the conditional statement below?
"If tomorrow is Monday, then today is Sunday."

- A. The inverse and contrapositive are true but the statement and converse are false.
- B. The statement, converse, inverse, and contrapositive are all true.
- C. The converse and inverse are true but statement and contrapositive are false.
- D. The statement and contrapositive are true but the inverse and converse are false.

Inverse → Both hypothesis and conclusion are Negated.

Converse → Hypothesis and conclusion switched

Contrapositive → Hypothesis & conclusion of Converse statement are negated

Statement: "If tomorrow is Monday then today is Sunday"

Sunday is always the day before Monday ⇒ TRUE

Inverse: "If tomorrow is NOT Monday then today is NOT Sunday" ⇒ TRUE

Converse: "If today is Sunday then tomorrow is Monday" ⇒ TRUE

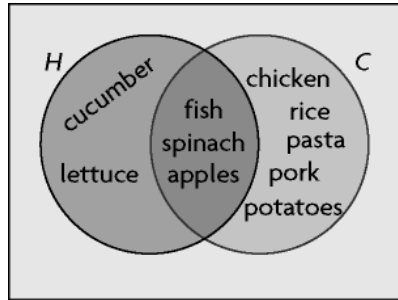
Contrapositive: "If today is NOT Sunday then tomorrow is NOT Monday" ⇒ TRUE

D

3. Which statement is true?

- A. The English language and the French language are disjoint sets. *No. There are some words in common. (café, bouquet)*
- B. Hockey equipment and lacrosse equipment are disjoint sets. *No. example: gloves → lacrosse goalie*
- C. Band instruments and orchestral instruments are disjoint sets. *No. example: drums*
- D.** Linear equations and quadratic equations are disjoint sets. *YES. An equation cannot be both linear and quadratic.*

Consider the following Venn diagram of foods we eat raw or cooked.
Use the diagram to answer questions #4-6



D

4. Determine $n(H \cap C)$.

- A. 2
- B. 5
- C. 11
- D. 3**

This means find the number of elements in set H AND C. (This is the overlapping section)

*fish
spinach
apples } 3 elements*

D

5. Determine $H \cap C$.

- A. {fish, spinach, apples, cucumber, lettuce, chicken, pork, rice, pasta, potatoes}
- B. {chicken, pork, rice, pasta, potatoes}
- C. {cucumber, lettuce}
- D.** {fish, spinach, apples}

This means list the subset of elements that are elements of set H AND C.

C

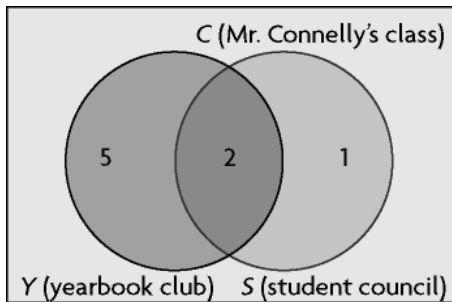
6. Determine $n(H \cup C)$.

- A. 2
- B. 5
- C. 10**
- D. 3

This means find the number of elements that are in set H OR C.
(which is all of them)

C

7. There are 28 students in Mr. Connelly's Grade 12 mathematics class. The number of students in the yearbook club and the number of students on student council are shown in the Venn diagram. Use the diagram to answer the following question.



How many students are in at least one of the yearbook club or on student council?

- A. 2
- B. 5
- C. 8**
- D. 7

Yearbook, Student Council, or both
 $5 + 1 + 2 = 8$

A

8. Consider the following two sets:

- $C = \{-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10\}$
- $B = \{-9, -6, -3, 0, 3, 6, 9, 12\}$

Determine $n(C \cap B)$.

- A. 3**
- B. 8
- C. 11
- D. 19

This means find the number of elements that are elements of set C AND D.

C

9. Consider the following two sets:

• $C = \{-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10\}$

• $B = \{-9, -6, -3, 0, 3, 6, 9, 12\}$

Determine $C \cup B$. \rightarrow List the set of elements that are elements of set C OR B
-7 is not in C or B

A. $\{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ X *7 is not in either set*

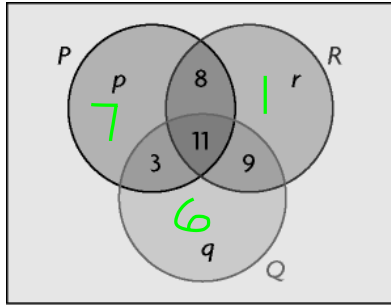
B. $\{-10, -9, -8, -6, -4, -3, -2, 0, 2, 3, 4, 6, 7, 9, 10\}$ X *7 is not in either set*

C. $\{-10, -9, -8, -6, -4, -3, -2, 0, 2, 3, 4, 6, 8, 9, 10, 12\}$ ✓

D. $\{-10, -9, -7, -6, -4, -3, -2, 0, 2, 3, 4, 6, 7, 9, 10, 12\}$ X *-7 is not in either set B or C.*

C

10. The three circles in the Venn diagram (P , Q , and R) contain the same number of elements. Which set of values is true for p , q , and r ?

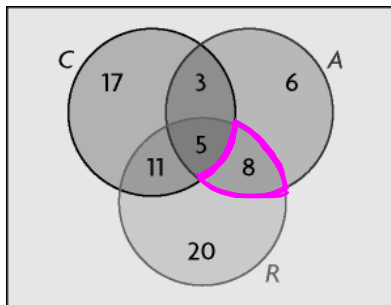


use options below & trial/error to find the values that result in each circle having the same number of elements.

- A. $p = 11, q = 11, r = 5$
- B. $p = 7, q = 8, r = 2$
- C.** $p = 7, q = 6, r = 1$
- D. $p = 14, q = 13, r = 7$

D

11. A summer camp offers canoeing, rock climbing, and archery. The following Venn diagram shows the types of activities the campers like.



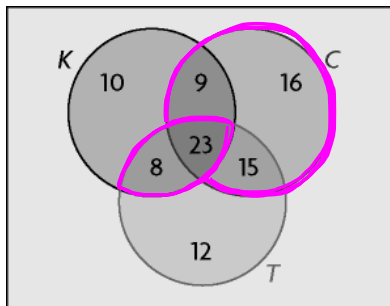
Use the diagram to determine $n((R \cap A) \setminus C)$.

- A. 14
- B. 5
- C. 26
- D.** 8

Find the number of elements that are in both sets R AND A but not C.

A

12. A restaurant offers Chinese, Thai, and Korean food. The following Venn diagram shows the types of food the customers like.



Use the diagram to determine $n((T \cap K) \cup C)$.

- A. 71
- B. 47
- C. 55
- D. 93

Find the number of elements in (T AND K) OR C.

* Make sure you don't count 23 twice.

$$8 + 23 + 9 + 15 + 16 = 71$$

B

13. Which sentence is written as a conditional statement?

- A. You can fool some of the people some of the time but you can't fool me.
- B. If you are a farmer, then you live in the country.
- C. If you can't beat them, join them.
- D. You get wet if you stand in the rain.

Remember: Conditional statements are "If - Then" statements

14. Which conditional statement is biconditional?

Biconditional means both the conditional statement and converse is true. (converse is when the hypothesis and conclusion are switched)

- A. If today is Labour Day, then it is November 3.
- B. If it is October, then students are in school.
- C. If today is Friday, then tomorrow is Saturday.
- D. If there is deep snow outside, then the outside temperature is below freezing.

*Note: For A, B & D the converse statements are also FALSE.

A: Conditional statement is FALSE (Labour Day is in September)
 B: Conditional statement is FALSE (what about weekends/holidays/evenings)
 C: Both are TRUE. (Saturday follows Friday. And Friday before Saturday)
 D: Conditional Statement is False (Snow could be days old & currently melting)

D

15. Which biconditional statement is false?

- A. 12 o'clock is midnight if and only if it is not noon.
- B. It is real maple syrup if and only if the syrup is made from maple sap.
- C. It is Valentine's Day if and only if it is February 14.
- D. It is a whale if and only if it is a mammal that lives in the ocean.

A: "If 12 o'clock is midnight, then it is not noon" } Both True
 "If it is not noon, then 12 o'clock is midnight"
 B: "If it is real maple syrup then it is made from maple sap." } Both TRUE
 "If it is made from maple sap then it is real maple syrup"
 C: "If it is Valentine's Day, then it is Feb 14" } Both TRUE
 "If it is Feb 14, then it is Valentine's Day"
 D: "If it is a whale then it is a mammal that lives in the Ocean." } FALSE ! Dolphins are also mammals
 "If it is a mammal that lives in the ocean then it is a whale!"

C

16. What is a contrapositive statement?

- A. a conditional statement in which the hypothesis and the conclusion are switched *This is a Converse*
- B. a statement that is formed by negating both the hypothesis and the conclusion of a conditional statement *This is the Inverse*
- C.** a statement that is formed by negating both the hypothesis and the conclusion of the converse of a conditional statement
- D. a statement that is formed by inverting both the hypothesis and the conclusion of a conditional statement *(Not a logical statement)*

C

17. Which statement is the converse of the conditional statement below?

- "If tomorrow is Monday, then today is Sunday." *Switch hypothesis and conclusion*
- A. If tomorrow is Sunday, then today is not Monday.
 - B. If tomorrow is not Monday, then today is not Sunday. *This is the inverse*
 - C.** If today is Sunday, then tomorrow is Monday.
 - D. If today is not Sunday, then tomorrow is not Monday. *This is the contrapositive*

A

18. What is true about the conditional statement below?
"If a bird has wings, then the bird can fly."

- A.** The converse and inverse are true but the statement and contrapositive are false.
- B. The statement and contrapositive are true but the inverse and converse are false.
- C. The statement, converse, inverse, and contrapositive are all true.
- D. The statement and inverse are true but the converse and contrapositive are false.

Statement: FALSE → Penguins
Converse: TRUE
Inverse: TRUE
Contrapositive: FALSE → Penguins

Short Answer

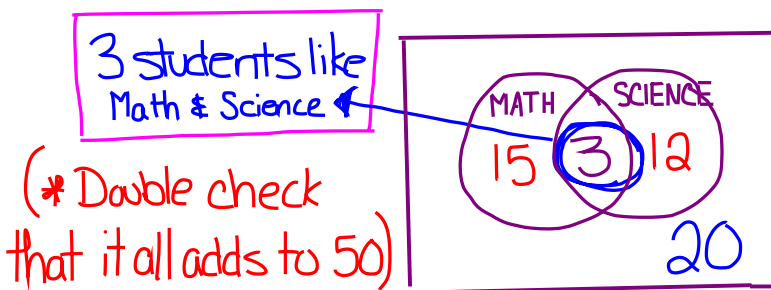
19. Carlos surveyed 50 students about their favourite subjects in school. He recorded his results.

Favourite Subject	Number of Students
mathematics	18
science	15
neither mathematics nor science	20

→ The Math circle should add to 18

Draw Venn Diagram start here

Determine how many students like mathematics and science.



There are 50 students all together so if 20 don't like either, that means 30 like either Math OR science.

But 18 + 15 = 33 → So 3 must have been counted twice

20. A music school offers lessons on 12 different instruments.

- | | | |
|----------|------------|-----------|
| piano | bagpipe | guitar |
| recorder | accordion | clarinet |
| violin | flute | xylophone |
| trumpet | steel drum | banjo |

Determine the number of instruments that are played with sticks or have a mouthpiece ($P \cup M$).

Green = Sticks
Pink = mouthpiece

$$n(P \cup M) = 7$$

(there are 7 instruments played with either sticks or mouthpiece)

21. Flightless birds include the ostrich, emu, penguin, and kiwi. Arctic birds include the snow goose, Arctic tern, osprey, penguin, and red-tailed hawk. Determine the union and intersection of these two sets.

Union = OR $F \cup B = \{ \text{Ostrich, emu, penguin, kiwi, snow goose, arctic tern, osprey, red-tailed hawk} \}$

Intersection = AND $F \cap B = \{ \text{Penguin} \}$

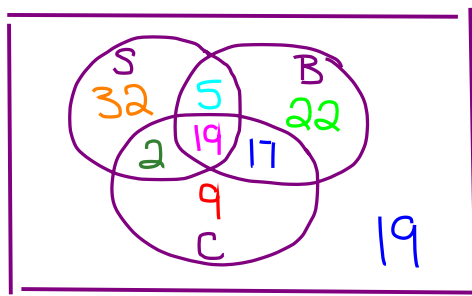
22. Grade 12 students were surveyed about their extra-curricular activities. There are 125 grade 12 students.

- Step 7 • 58 belonged to a sports team (S)
- Step 6 • 63 belonged to a band (B)
- Step 5 • 47 belonged to a school club (C)
- Step 4 • 24 belonged to a sports team and a band
- Step 3 • 21 belonged to a sports team and a school club
- Step 2 • 36 belonged to a band and a school club
- Step 1 • 19 engaged in all three activities ← start here: Step 1

Use a Venn Diagram to represent the information.

How many students belong to both a sports team and a band but not a school club?

Write your answer in set notation.



- Step 2: $36 - 19 = 17$
- Step 3: $21 - 19 = 2$
- Step 4: $24 - 19 = 5$
- Step 5: $47 - 2 - 19 - 17 = 9$
- Step 6: $63 - 5 - 19 - 17 = 22$
- Step 7: $58 - 2 - 19 - 5 = 32$

We know there are 125 students. So to complete the diagram it should all add to 125.

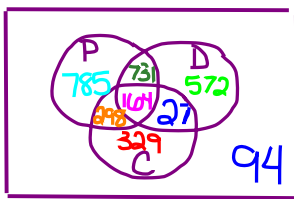
Sports AND band but not club

$$n((S \cap B) \setminus C) = 5$$

23. The city surveyed 3000 people about how they travel to work.

- Step 7 • 1978 took public transit (P)
- Step 6 • 1494 drove (D)
- Step 5 • 818 cycled (C) *The cycle circle should add to 818.*
- Step 4 • 731 took public transit and drove only
- Step 3 • 298 took public transit and cycled only
- Step 2 • 27 drove and cycled only (*only → not public transit*)
- Step 1 • 164 used all three modes of transportation ← *Start here.*

How many people use public transit only? Use a Venn diagram to show your answer.



Step 5:
 $818 - 298 - 164 - 27 = 329$

Step 6:
 $1494 - 731 - 164 - 27 = 572$

Step 7:
 $1978 - 731 - 164 - 298 = 785$

$n(\text{Public Transit Only})$
 $= n(P/D/C) = 785$

Remember 3000 people were surveyed. That means 94 people don't use transit, bikes, or cars.

24. Write the converse of the conditional statement below.
 "If you make unleavened bread, then you make bread without yeast."

Converse \Rightarrow switch hypothesis & conclusion

"If you make bread without yeast, then you make unleavened bread."

25. If the statement below is biconditional, rewrite it in biconditional form. If the statement is not biconditional, provide a counterexample.

"If you have Canadian coin worth two dollars, then you have a toonie."

A statement is biconditional if both the statement and the converse are true.

Statement: TRUE (all Canadian \$2 coins are toonies)

Convers: "If you have a toonie, then you have a Canadian coin worth two dollars." TRUE (toonies are only used to describe \$2 coins)

So the statement is biconditional.

"You have a Canadian coin worth two dollars if and only if you have a toonie."

Converse = switch hypothesis & conclusion

26. Write the converse of the conditional statement below. Verify the converse or disprove it with a counterexample.

"If an animal has eight legs, then the animal is a scorpion."

Converse : "If an animal is a scorpion, then the animal has eight legs."

(true \rightarrow scorpions have eight legs)

Problem

27. A restaurant survey asked 300 people if they preferred Indian or Chinese food.

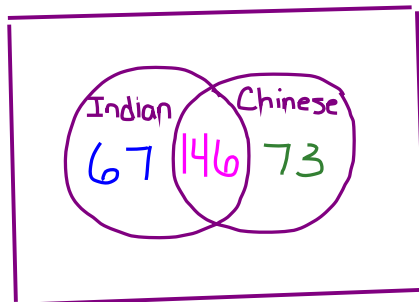
• 146 people liked both. \leftarrow start here - step 1

Step 2 • 213 people liked Indian food.

Step 3 • 219 people liked Chinese food.

Determine how many people did not like Indian or Chinese food.

Draw a Venn diagram to show your solution.



Step 2: The Indian Food circle must total 213. $(213 - 146 = 67)$

Step 3: The Chinese Food circle must total 219. $(219 - 146 = 73)$

Step 4: 300 people were surveyed so the Venn Diagram must add to 300. $(300 - 67 - 146 - 73 = 14)$

14 people did not like either food

28. Consider this statement: "You are a juror if and only if you were chosen to be on a jury."

a) Write a conditional statement and its converse.

Conditional: "If you are a juror, then you were chosen to be on a jury."

Converse: "If you were chosen to be on a jury, then you are a juror."

b) Are the statements you wrote in part a) true or false? Explain how you know.

Conditional is true \rightarrow there is only one way to be on a jury.

Converse is true \rightarrow if you are chosen, then you are on the jury.

c) Is the original statement true or false? Explain how you know.

The original biconditional statement is true because the conditional & converse are true.

29. Consider this conditional statement:

"If $\sqrt{b} = a$, then $a < b$."

a) Write the converse, the inverse, and the contrapositive.

Converse: "if $a < b$, then $\sqrt{b} = a$ "

inverse: "if $\sqrt{b} \neq a$, then $a \geq b$."

Contrapositive: "if $a \geq b$, then $\sqrt{b} \neq a$."

b) Verify that each statement is true, or disprove it with a counterexample.

Converse: FALSE Counter example: $a=1, b=4$
($1 < 4$ but $\sqrt{4} \neq 1$)

inverse: FALSE counter example: $a=1, b=4$
($\sqrt{4} \neq 1$, but $a < b$)

Contrapositive: FALSE counter example $a=\frac{1}{2}, b=\frac{1}{4}$
($\frac{1}{2} \geq \frac{1}{4}$, but $\sqrt{\frac{1}{4}} = \frac{1}{2}$)